Christ Church Grammar School

Semester One Examination, 2016

Question/Answer Booklet

MATHEMATICS METHODS UNIT 3

Section One: Calculator-free

If required by your examination administrator, please place your student identification label in this box

Student Number: In figures | | | | | | | | |

In words _____

Your name _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time for section: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	48	35
Section Two: Calculator-assumed	12	12	100	92	65
			Total	140	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

See next page

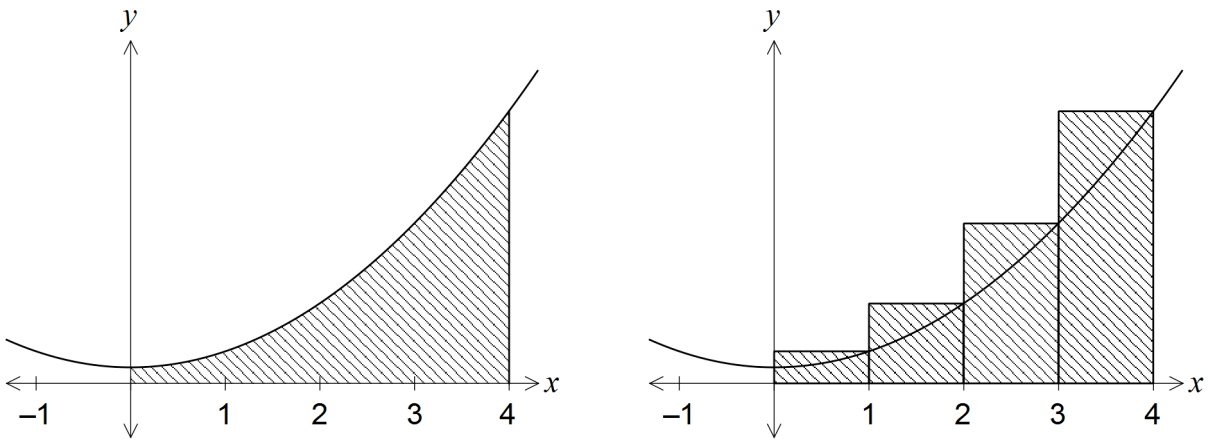
Section One: Calculator-free **35% (48 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 1 **(5 marks)**

Part of the graph of $y = x^2 + 1$ is shown in the diagrams below.



An approximation for the area beneath the curve between $x = 0$ and $x = 4$ is made using rectangles as shown in the right-hand diagram. Determine the exact amount by which the approximate area exceeds the exact area.

Question 2 **(9 marks)**

(a) Differentiate the following with respect to x, simplifying your answers.

(i) $y = \int_x^1 (t - t^3)\, dt.$ (2 marks)

(ii) $y = \sin^3(2x + 1).$ (3 marks)

Question 2 (continued)

(b) Determine the values of the constants a, b and c, given that $f''(x) = e^{3x}\left(ax^2 + bx + c\right)$ when $f(x) = x^2e^{3x}$. (4 marks)

Question 3 **(7 marks)**

Consider the function defined by $f(x) = \frac{x}{2} - \sqrt{x},\ x \geq 0$.

(a) Determine the coordinates of the stationary point of $f(x)$. (3 marks)

(b) Use the second derivative test to determine the nature of the stationary point found in (a). (3 marks)

(c) State the global minimum of $f(x)$. (1 mark)

Question 4 **(5 marks)**

The area of a segment with central angle θ in a circle of radius r is given by $A = \frac{r^2}{2}(\theta - \sin\theta)$.

Use the increments formula to find the approximate increase in area of a segment in a circle of radius 10 cm as the central angle increases from $\frac{\pi}{3}$ to $\frac{11\pi}{30}$.

See next page

Question 5 **(5 marks)**

(a) Differentiate $y = \dfrac{2x+1}{e^x}$, simplifying your answer. (3 marks)

(b) Using the result in (a) or otherwise, evaluate $\int_1^2 \left(\dfrac{1-2x}{e^x} \right) dx$. (2 marks)

See next page

Question 6 **(6 marks)**

The discrete random variable X has the probability distribution shown in the table below.

x	0	1	2	3
$P(X = x)$	$\frac{2a^2}{3}$	$\frac{1-3a}{3}$	$\frac{1+2a}{3}$	$\frac{4a^2}{3}$

Determine the value of the constant a.

See next page

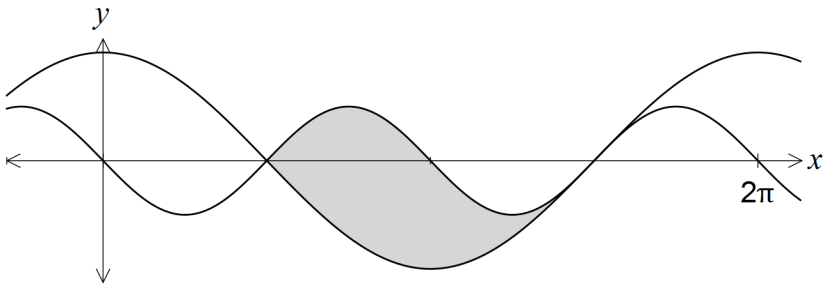
Question 7 **(5 marks)**

The area bounded by the curve $y = e^{2-x}$ and the lines $y = 0$, $x = 1$ and $x = k$ is exactly $e - 1$ square units. Determine the value of the constant k, given that $k > 1$.

See next page

Question 8 **(6 marks)**

The shaded region on the graph below is enclosed by the curves $y = -\sin(2x)$ and $y = 2\cos x$.



(a) Given that $\sin(2x) = 2\sin x \cos x$, show that the first two roots of the equation $2\cos x = -\sin(2x)$ are $\frac{\pi}{2}$ and $\frac{3\pi}{2}$. (3 marks)

(b) Hence find the area of the enclosed region in the diagram above. (3 marks)

End of questions

Additional working space

Question number: _____

Christ Church Grammar School

Semester One Examination, 2016

Question/Answer Booklet

MATHEMATICS METHODS UNIT 3

Section Two: Calculator-assumed

If required by your examination administrator, please place your student identification label in this box

Student Number: In figures | | | | | | | | |

In words _____

Your name _____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time for section: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators approved for use in the WACE examinations

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See next page

Section Two: Calculator-assumed **65% (92 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

Question 9 **(4 marks)**

A recent news report said that it took 34 months for the population of Australia to increase from 23 to 24 million people.

(a) Assuming that the rate of growth of the population can be modelled by the equation $\frac{dP}{dt} = kP$, where P is the population of Australia at time t months, determine the value of the constant k. (2 marks)

(b) Assuming the current rate of growth continues, how long will it take for the population to increase from 24 million to 25 million people? (2 marks)

Question 10 **(7 marks)**

A small object is moving in a straight line with acceleration $a = 6t + k$ ms^{-2}, where t is the time in seconds and k is a constant. When $t = 1$ the object was stationary and had a displacement of 4 metres relative to a fixed point O on the line. When $t = 2$ the object had a velocity of 1 ms^{-1}.

(a) Determine the value of k and hence an equation for the velocity of the object at time t. (4 marks)

(b) Determine the displacement of the object when $t = 2$. (3 marks)

See next page

Question 11 **(7 marks)**

It is known that 15% of Year 12 students in a large country study advanced mathematics.

A random sample of n students is selected from all Year 12's in this country, and the random variable X is the number of those in the sample who study advanced mathematics.

(a) Describe the distribution of X. (2 marks)

(b) If $n = 22$, determine the probability that

(i) three of the students in the sample study advanced mathematics. (1 mark)

(ii) more than three of the students in the sample study advanced mathematics. (1 mark)

(iii) none of the students in the sample study advanced mathematics. (1 mark)

(c) If ten random samples of 22 students are selected, determine the probability that at least one of these samples has no students who study advanced mathematics. (2 marks)

Question 12 **(8 marks)**

The height of grain in a silo, initially 0.4 m, is increasing at a rate given by $h'(t) = 0.55t - 0.05t^2$ for $0 \le t \le 11$, where h is the height of grain in metres and t is in hours.

(a) At what time is the height of grain rising the fastest? (2 marks)

(b) Determine the height of grain in the silo after 11 hours. (3 marks)

(c) Calculate the time taken for the grain to reach a height of 4.45 m. (3 marks)

See next page

Question 13 **(5 marks)**

Records of a company that has a large workforce indicate that 35 percent of employees take sick leave during any given year.

(a) If the records of five employees are selected at random from the previous year, what is the probability that fewer than three took sick leave? (2 marks)

Amongst the 20 management staff of the company, seven of them had taken sick leave during the previous year.

(b) If five management staff are selected at random, what is the probability that two or less took sick leave during the previous year? (3 marks)

See next page

Question 14 **(14 marks)**

(a) Determine the mean of a Bernoulli distribution with variance of 0.24. (3 marks)

(b) A Bernoulli trial, with probability of success p, is repeated n times. The resulting distribution of the number of successes has an expected value of 5.76 and a standard deviation of 1.92. Determine n and p. (4 marks)

See next page

Question 14 (continued)

(c) The probability that a student misses his bus to school is 0.2, and the probability that he misses the bus on any day is independent of whether he missed it on the previous day.

Over five consecutive weekdays, what is the probability that the student

(i) only misses the bus on Tuesday? (2 marks)

(ii) misses the bus at least twice? (2 marks)

(iii) misses the bus on Tuesday and on two other days? (3 marks)

See next page

Question 15 **(9 marks)**

A particle moves in a straight line according to the function $x(t) = \frac{t^2 + 3}{t + 1}$, $t \geq 0$, where t is in seconds and x is the displacement of the particle from a fixed point O, in metres.

(a) Determine the velocity function, $v(t)$, for the particle. (2 marks)

(b) Determine the displacement of the particle at the instant it is stationary. (2 marks)

(c) Show that the acceleration of the particle is always positive. (2 marks)

See next page

Question 15 (continued)

(d) After five seconds, the particle has moved a distance of k metres.

(i) Explain why $k \neq \int_0^5 v(t)\,dt$. (1 mark)

(ii) Calculate k. (2 marks)

Question 16 **(8 marks)**

The discrete random variable Y has the probability distribution shown in the table below.

y	-2	-1	0	1	2
$P(Y = y)$	0.4	0.2	0.1	0.1	0.2

(a) Determine $P(Y \geq 0 \mid Y \leq 1)$. (2 marks)

(b) Calculate

(i) $E(Y)$. (2 marks)

(ii) $E(1-2Y)$. (1 mark)

(c) Calculate

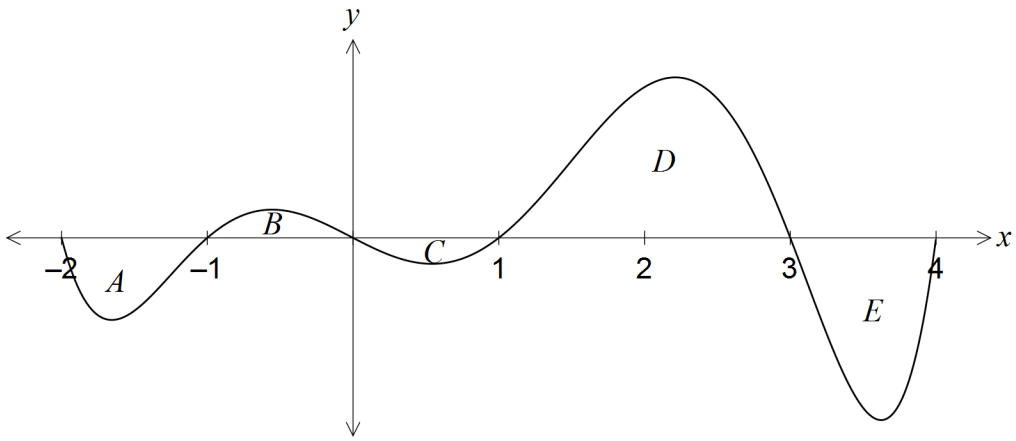
(i) $Var(Y)$. (2 marks)

(ii) $Var(1-2Y)$. (1 mark)

See next page

Question 17 **(8 marks)**

The graph of the function $y = f(x)$ is shown below for $-2 \le x \le 4$.



The area of regions enclosed by the *x*-axis and the curve, *A*, *B*, *C*, *D* and *E*, are 12, 7, 5, 32 and 21 square units respectively.

(a) Determine the value of $\int_{-2}^{4} f(x)\,dx$. (2 marks)

(b) Determine the area of the region enclosed between the graph of $y = f(x)$ and the *x*-axis from $x = 0$ to $x = 4$. (2 marks)

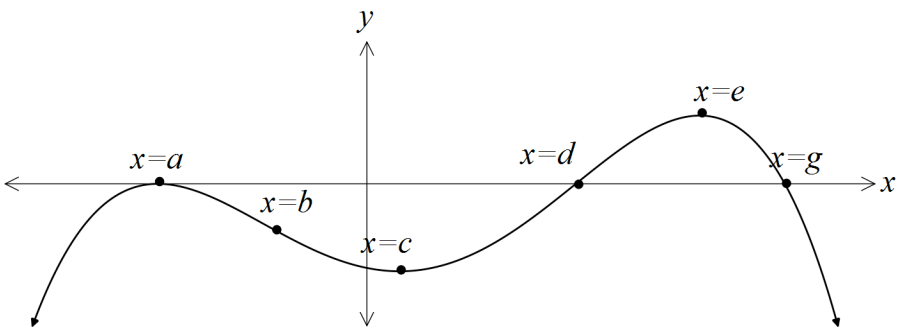
(c) Determine the values of

(i) $\int_{0}^{3} f(x) + 3\,dx$. (2 marks)

(ii) $2\int_{0}^{1} f'(x)dx$. (2 marks)

Question 18 **(8 marks)**

The graph of $y = f'(x)$, the derivative of a polynomial function *f*, is shown below. The graph of $y = f'(x)$ has stationary points when $x = a$, $x = c$ and $x = e$, points of inflection when $x = b$ and $x = d$, and roots when $x = a$, $x = d$ and $x = g$, where $a < b < c < d < e < g$.



(a) For what value(s) of *x* does the graph of $y = f(x)$ have a point of inflection? (1 mark)

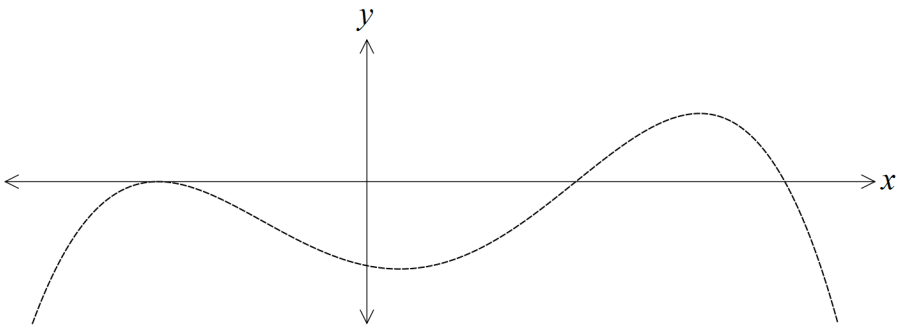
(b) Does the graph of $y = f(x)$ have a local maximum? Justify your answer. (2 marks)

See next page

Question 18 (continued)

(c) Does the graph of $y = f(x)$ have a horizontal point of inflection? Justify your answer. (2 marks)

(d) On the axis below, sketch a possible graph of $y = f''(x)$. The graph of $y = f'(x)$ is shown with a broken line for your reference. (3 marks)



Question 19 **(7 marks)**

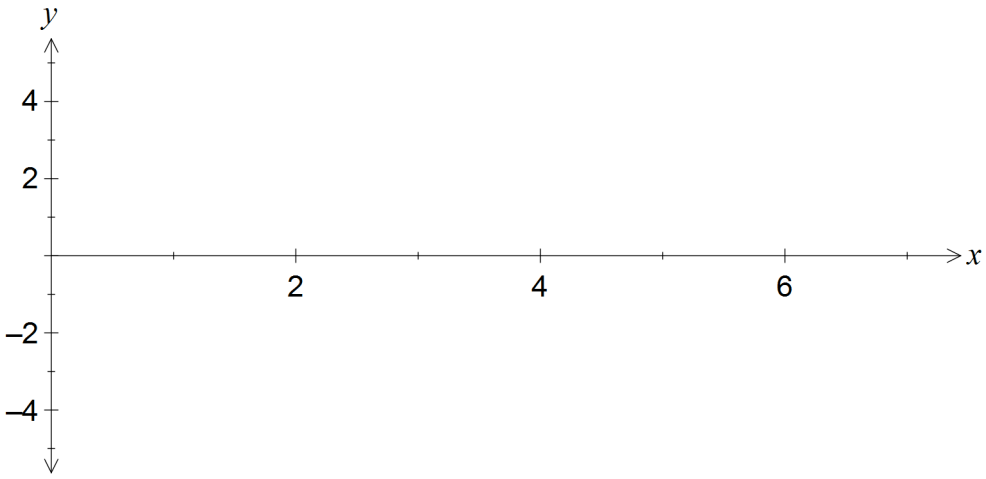
Consider the function $f(t) = \frac{t-4}{2}$ and the function $A(x) = \int_0^x f(t)\,dt$.

(a) Complete the table below. (2 marks)

x	0	1	2	3	4	5	6
$A(x)$	0	-1.75					

(b) For what value(s) of x is the function $A(x)$ increasing? (1 mark)

(c) On the axes below, sketch the graph of $y = A(x)$ for $0 \le x \le 6$. (2 marks)



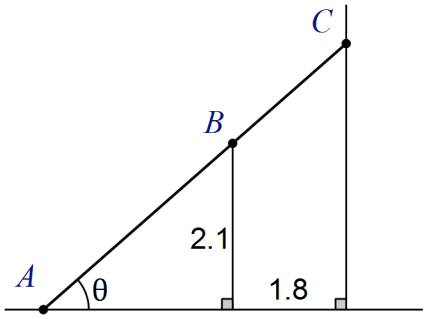
(d) Determine

(i) when $A'(x) = 0$. (1 mark)

(ii) the function $A(x)$ in terms of x. (1 mark)

Question 20 **(7 marks)**

A vertical wall, 2.1 metres tall, stands on level ground and 1.8 metres away from the wall of a house. A ladder, of negligible width, leans at an angle of θ to the ground and just touches the ground, wall and house, as shown in the diagram.



(a) Show that the length of the ladder, L, is given by $L = \frac{2.1}{\sin\theta} + \frac{1.8}{\cos\theta}$. (3 marks)

(b) Use a calculus method to determine the length of the shortest ladder that can touch the ground, wall and house at the same time. (4 marks)

End of questions

Additional working space

Question number: _____